Extracting conditional densities from swaption prices via Breeden-Litzenberger

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1 Introduction

Swaptions are one of the most traded instruments within the interest rate derivatives market. Moreover, they provide information about the swap rate along a wide maturity spectrum from 1 year to 30 years. Therefore, these contracts provide rich information about market expectations of rates and volatility.
Swaptions give holders the option to enter into an interest rate swap, with payer swaptions paying a fixed rate and receiving the floating swap rate, and receiver swaptions paying the floating swap rate and receiving the fixed rate. The forward swap rate is determined such that the present value of the swap is equal to 0.
As it can be seen from the formula below, payer swaptions (P_m, n) can be interpreted as call options on the swap rate, while by the same logic receiver swaptions (R_m, n) are put options on the swap rate.

$$\mathcal{P}_{m,n}(t,K) = A_{m,n}(t)E_t^A\left[(S_{m,n}(T_m) - K)^+\right]$$

$$\mathcal{R}_{m,n}(t,K) = A_{m,n}(t)\,E_t^A\left[(K - S_{m,n}(T_m))^+\right]$$

Where A denotes the annuity measure associated with using $A_{m,n}(t)$ as numeraire, such that the forward swap rate $S_{m,n}$ is a martingale.
This project aims to extract information on market expectations regarding changes in the policy rate of the Federal Reserve (Fed). We do so by conducting a very simplified event-study inspired research, extracting probability densities around important Federal Open Market Committee (FOMC) decision dates by using the results of Breeden-Litzenberger (1978), and looking at the changes to the distribution and the first four moments.
The main contribution of this project is primarily methodological, the results should be interpreted in light of the limited data availability.
The replication code and the dataset are provided through a GitHub repository in the References section.

2 Methodology

The goal of this project is to study the effect of FOMC decisions on market uncertainty by building on the findings of Schwartz and Trolle (2014). The authors extract conditional swap rate moments, investigate the linkages between the moments, and construct synthetic variance- and skewness swaps to study associated risk premia.

While we follow the core idea of the paper (the use of swaptions to extract market information), instead of estimating conditional moments of the forward swap rate through static replication, this paper computes conditional densities of the swap rate on the day before, the day of, and the day after important FOMC decisions.

This choice was motivated by the limited availability of data (the strike grid employed for the estimations consisted of 9 data points). Using the Carr-Madan style static replication to obtain conditional moments would mean relying more strongly on deeply out-of-the-money (OTM) option prices, which would have had to be interpolated from the few strikes that are provided via the Bloomberg terminal.

Instead, this paper used the Breeden-Litzenberger formula, which under certain assumptions, as detailed below, allows for the recovery of densities even with such a sparse strike grid.

To calculate payer swaption (call) prices, we input implied volatilities to the Bacheleir formula.

$$C = e^{-rT}\left[(F-K)\Phi(d) + \sigma\sqrt{T}\,\phi(d)\right]$$

$$d = \frac{F-K}{\sigma\sqrt{T}}$$

The forward rate F was set to 0 for tractability. Therefore, all the results below should be interpreted in relative terms.

2.1 Density estimation

To estimate the densities, we apply the logic of the findings of Breeden - Litzenberger, namely:

$$\frac{\partial^2 C\big(F_0^T, K, T\big)}{\partial K^2} = \varphi\big(F_T, T;\, F_0^T, 0\big)\,\Big|_{F_T=K}$$

to infer the risk-neutral probability density of swaption rates from swaption prices.

A possible approach would have been the interpolation of the call price curve from the available data points using cubic splines and quadratic smoothing, but this proved to be highly unstable and resulted in oscillatory and/or negative probabilities. Therefore, this paper uses discretised second derivatives from call prices which have been adjusted to ensure no arbitrage conditions.

This has been done the following way: let S_i denote the slope of the call price function between strikes K_{i-1} and K_i. Then the atomic probability at strike K_i is given by: $p_i = S_i - S_{i-1}$
To ensure no-arbitrage, the following assumptions were enforced: (i) call prices are monotonous and decreasing in strike, they are (ii) convex in strike and (iii) a lower arbitrage bound of $\frac{\partial C}{\partial K} \geq -1$ is applied.

2.2 Moment estimation, visualisation

The moments reported in the tables were computed directly from the atomic densities, and therefore are not affected by any kind of smoothing. For example, the mean for each day is calculated as a probability weighted average of the strikes, with p_i obtained as outlined above.
For easier visual interpretation, the continuous densities presented have been constructed using kernel-smoothing with the base of the Gaussian kernel set at 25bps. This choice was motivated by the following considerations: this bandwidth corresponds to the finest spacing in the strike grid, larger bandwidths would over-smooth the density, while smaller bandwidths would introduce spurious local features that are not supported by the data (such as artificial modalities).
However, because of the still significant amount of distortion introduced by kernel smoothing, the analysis relies on interpreting the moments, and the plots are only included for visualisation purposes.

3 Data

3.1 Swaption data

The data used for the project has been collected from the Bloomberg Swaption Manager. The primary source are normal (Bachelier) volatilities across the following grid of strikes: $\pm 200, 100, 50, 25$ bps and the ATM volatility from the Interest Rate Volatility Cube. While for certain dates the strikes ± 75 bps were also available, these were dropped from the dataset to ensure consistency across events.
The contracts used were 3Mx1Y USD payer swaptions, which can be interpreted as call options on the future swap rate with a 3 month expiry and 1 year maturity.
This combination of maturity and expiry has been chosen due to theoretical reasons and practical considerations: FOMC decisions mostly impact the front end of the yield curve, moreover, to isolate the effect of a monetary policy decision it is reasonable to take a swaption with the shortest possible expiry. As for the practical reasons, while 1Mx1Y swaptions also exist, their volatilities were not available across all chosen dates in the Bloomberg Interest Rate Volatility Cube.

3.2 FOMC events

The following FOMC decisions were analysed for the project: March 3rd 2020, March 15th 2020, March 16th 2022, June 15th 2022, July 27th 2022, September 1st 2022.
March 3 was the date of the first emergency rate cut due to Covid-19, while on March 15 the policy rate was further decreased to 0. March 16 2022 is the first rate hike, while June 15, July 27 and September 21 of 2022 were all chosen due to the intensity of the increase (all 75bps rate hikes).
In our results section we report the findings from the most relevant 3 events; March 3 and 16 2020 and June 15 2022.
The initial research hypothesis was that variance of the forward swap rate would decrease on the days following FOMC decisions, signalling that new information reduces market uncertainty.

4 Results

In the following tables and figures we report the summary statistics and implied distributions recovered from swaption prices with some short comments for the 3 most relevant events.
Overall, we did not find strong evidence of a systematic decline in option-implied future swap-rate variance following policy announcement dates. This may be due to anticipatory market pricing, the sparse strike grid available, and the choice of a 3-month to 1-year expiry–maturity window that may be ill-suited to capture policy effects.

4.1 Event 1 - March 3 2020

On March 3, 2020, the US Federal Reserve announced a historic emergency interest rate cut of 50bps in response to the COVID-19 pandemic. From the moments table we can observe that variance of the future swap rate decreases slightly post-announcement, potentially indicating that the FOMC decision decreased market uncertainty. Skewness is negative, suggesting downward moves (further rate cuts) were seen as more likely. Kurtosis is the highest for this event out of the 3 included in this analysis, suggesting that investors priced in higher tail risk — i.e., large moves in the future swap rate were seen as more likely than during subsequent events. This seems plausible given the emergency rate cut and the market turbulence in March 2020. However, it should be taken into consideration that the magnitude of all changes are small.

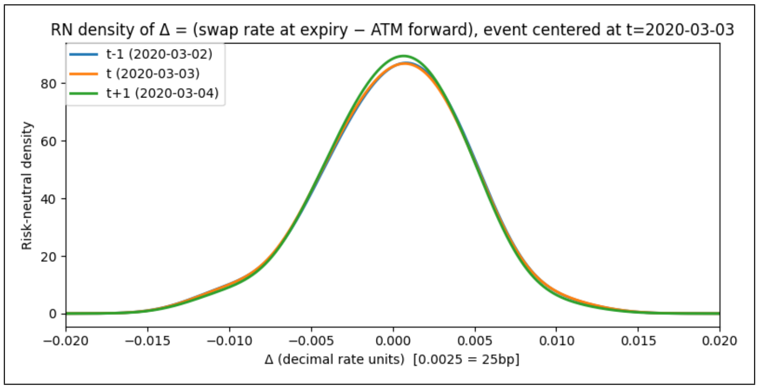

Figure 1: Risk-neutral density centred around March 3 2020

	t-1	t	t+1
Mean(Δ)	0.00015201	0.00012421	9.9069e-05
Variance	1.6288e-05	1.6282e-05	1.4859e-05
Skewness	-0.35327	-0.31681	-0.36509
Kurtosis	3.4092	3.3985	3.4863

Table 1: Conditional moments centred around March 3 2020

4.2 Event 2 - March 16 2022

The following event is March 16 2022, when the FOMC decided to increase interest rates for the first time since the start of the pandemic.
Again, variance decreases slightly post-announcement, potentially indicating that the FOMC decision decreased market uncertainty. Skewness remains positive throughout the event window and increases slightly post-announcement, signalling that the market was interpreting the event as the start of a hiking cycle. Kurtosis shows a modest increase from $t-1$ to t, suggesting the market briefly priced higher tail risk around the announcement itself, before declining at $t+1$ as the initial decision was absorbed. However, kurtosis remains elevated relative to a normal distribution throughout, reflecting persistent uncertainty about the pace of subsequent hikes. The magnitude of changes is still small.

	t-1	t	t+1
Mean(Δ)	-0.00038443	-0.00025412	-0.00022112
Variance	2.7571e-05	2.4789e-05	2.3561e-05
Skewness	0.1316	0.1186	0.12091
Kurtosis	2.6427	2.8032	2.8812

Table 2: Conditional moments centred around March 16 2022

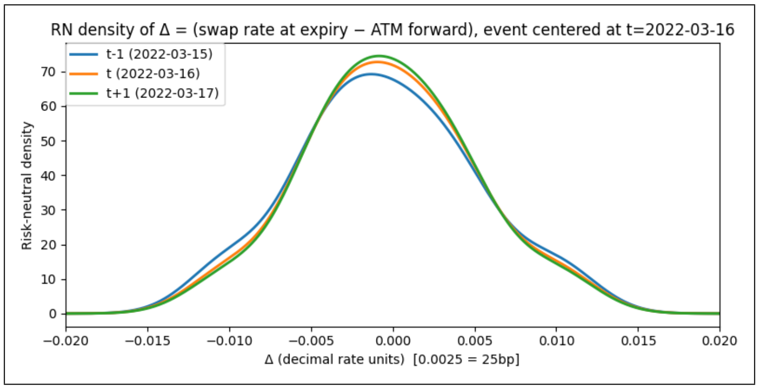

Figure 2: Risk-neutral density centred around March 16 2022

4.3 Event 3 - June 15 2022

The rate hike of June 15 was the biggest since 1994 (75bps).
Variance increased post-announcement, possibly indicating that the FOMC decision added uncertainty instead of resolving it.
This counter-intuitive result suggests that the expectations of the market were different in terms of either the magnitude of the hike or the accompanying forward guidance, leading to greater dispersion in views about the future rate trajectory. Kurtosis remains relatively low throughout the event window, indicating that the observed increase in uncertainty primarily reflects broader dispersion of outcomes rather than heightened tail risk. The distribution becomes more spread out overall.
Skewness remains positive and increases from t to $t+1$, signalling that the market was still expecting further rate hikes and viewing the distribution of future rates as asymmetrically skewed toward higher levels. The mean of the distribution shifts upward, reflecting a repricing of the expected rate path, though the magnitude of changes in higher moments—particularly the variance increase—dominates the interpretation. However, the magnitude of changes is still small.

	t-1	t	t+1
Mean(Δ)	-0.00032597	-0.00033373	-0.00038632
Variance	3.6051e-05	3.5506e-05	3.7304e-05
Skewness	0.068516	0.071494	0.078765
Kurtosis	2.2126	2.2361	2.1625

Table 3: Conditional moments centred around June 15 2022

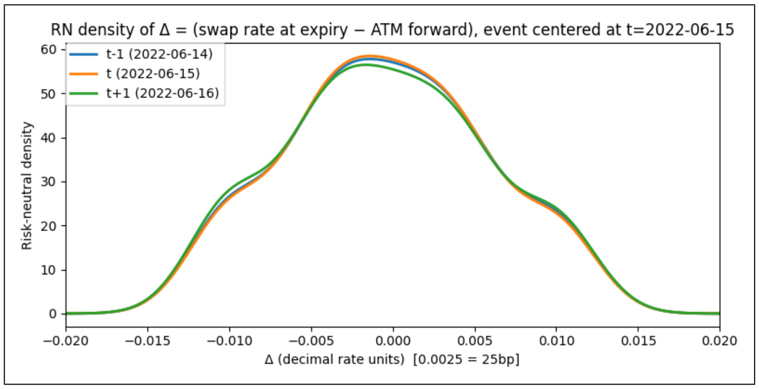

Figure 3: Risk-neutral density centred around June 15 2022

5 Limitations

This research has numerous limitations, which are important to highlight. First, the problem of data availability significantly curtailed the methods at disposal and should also be taken into consideration when interpreting the findings.
Further research could improve on this by expanding the strike space outside of the ± 200 bps region, and making the strike grid finer could also enhance the stability of results. In case of a fine enough strike grid the discretised differentiation could be replaced by the original Breeden-Litzenberger method.
Moreover, there is a temporal mismatch between the derivatives used for probability estimation and the time window of the event study. The 3-month swaptions reflect market expectations about swap rates three months in the future, not the immediate event impact. If multiple FOMC meetings occur within this window, their effects cannot be disentangled, and even with a single event, the analysis captures changes in medium-term expectations rather than direct impact. Additionally, swap rates represent expectations about the average path of short-term rates over multiple years, creating further separation from the Fed's actual policy rate. A cleaner event study would require shorter-dated derivatives more directly linked to the policy rate, such as fed funds futures or overnight indexed swaps.
Regarding the assumptions in our density estimation, we normalise the probabilities such that they sum to one, implicitly assuming that the entire probability mass of the risk-neutral distribution lies within the quoted strike range of ± 200 basis points.
While this is unlikely to hold exactly, as some tail probability may lie outside the strike interval, this does not create mathematical inconsistencies in our estimation procedure.

6 Concluding remarks

Taking into account the limitations described above, the added value of this research lies in demonstrating a complete methodology for extracting risk-neutral densities from swaption markets using the Breeden-Litzenberger formula. While prior literature has predominantly applied this approach to caps, floors, or shorter-dated interest rate derivatives, we show that the method can be effectively implemented using swaptions with standard no-arbitrage assumptions (call prices are convex and monotone-decreasing in strike). This paper shows that even with discrete strike grids typical of OTC swaption markets, these constraints can be enforced through straightforward convex optimization, making the approach accessible for empirical implementation.

The use of swaption contracts offers several practical advantages: they are among the most liquid interest rate derivatives, particularly at medium-term expiries, providing robust market-implied distributions even during volatile periods; they reflect expectations about the entire term structure of rates rather than isolated maturities; and their pricing embeds information about multiple forward rate paths, capturing the market's view on how monetary policy will evolve over extended horizons. This makes swaptions particularly well-suited for studying how policy announcements reshape not just immediate rate expectations, but the entire forward curve.

The framework we developed could be extended to other central bank event studies or adapted to analyse how macroeconomic data releases alter the distribution of future rate paths across different markets and jurisdictions.

7 References

Breeden, Douglas T., and Robert H. Litzenberger. 1978. "Prices of State-Contingent Claims Implicit in Option Prices." *The Journal of Business* 51 (4): 621. https://doi.org/10.1086/296025.

Trolle, Anders B., and Eduardo S. Schwartz. 2014. "The Swaption Cube." *Review of Financial Studies* 27 (8): 2307–53. https://doi.org/10.1093/rfs/hhu015.

Access to the code and data: `https://github.com/vsophie2002-rgb/swaption_bl`